

What is really “quantum” in quantum theory?

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January 14, 2003

Abstract

By analysing probabilistic foundations of quantum theory we understood that the so called quantum calculus of probabilities (including Born’s rule) is not the main distinguishing feature of “quantum”. This calculus is just a special variant of a contextual probabilistic calculus. In particular, we analysed the EPR-Bohm-Bell approach by using contextual probabilistic models (e.g., the frequency von Mises model). It is demonstrated that the EPR-Bohm-Bell consideration are not so much about “quantum”, but they are merely about contextual. Our conjecture is that the “fundamental quantum element” is the Schrödinger evolution describing the very special dependence of probabilities on contexts. The main quantum mystery is neither the probability calculus in a Hilbert space nor the noncommutative (Heisenberg) representation of physical observables, but the Schrödinger evolution of contextual probabilities.

1 Introduction

Last years there was demonstrated increasing interest to foundations of quantum theory.¹ I would like to mention a few recent investigations on the gen-

¹Intensive development of quantum information theory and quantum computing as well as new experimental technologies are the main stimulating factors.

eral probabilistic structure of quantum theory, L. Ballentine [1], L. Hardy [2], R. Gill, G. Weihs, A. Zeilinger, M. Zukowski [3], S. Gudder [4], A. Khrennikov [5], I. Pitowsky [6], J. Summhammer [7] and on the EPR-Bell experiment, L. Accardi [8], W. De Baere [9], W. De Myunck [10], K. Hess and W. Philipp [11], A. Khrennikov [12], I. Volovich [13].

Roughly speaking all these investigations are devoted to the problem formulated in the title of this paper:

“Which element of quantum theory is really the fundamental “quantum element”?”

The Hilbert space representation of states? Born’s rule for probability and, in particular, interference of probabilities? Noncommutativity of observables? Heisenberg uncertainty relation? Bohr’s complementarity? Reduction? Schrödinger evolution? Or we could not at all split quantum theory into essential and nonessential elements? At the moment the latter point of view dominates in the quantum community: quantum theory as indivisible whole.²

I think that similar problems stimulated recent investigations of G. ‘t Hooft [15] who proposed a (discrete) classical deterministic model which beneath quantum mechanics. I remark that interference-like statistical effect for macroscopic particles (driven by classical electromagnetic forces) was numerically simulated by A. Khrennikov and Ja. Volovich [16] in the model with discrete time.

Mentioned investigations (besides ‘t Hooft’s model) are devoted to the probabilistic structure of quantum theory. This is very natural, since quantum theory is a **statistical theory**. This theory can not say anything about individual quantum systems (of course, it depends on an interpretation of quantum mechanics).

Creation of a mathematical theory that would describe dynamics of individual quantum systems is the greatest problem for physics of this century or even millennium.

In the present paper I would like to analyse the structure of quantum theory by using the so called contextual probabilistic model, see [5]. On the one hand, by such an analysis we can better understand the probabilistic structure of quantum theory. On the other hand, by comparing contextual and quantum models we can try to find the fundamental element of quantum theory.

²However, see S. Gudder [4] and C. Fuchs [14].

2 On exotic theories of probability

I would like to start with a rather provocative paper [17] of R. Gill, namely, with comments on the chapter *Khrennikov and exotic probabilities*. I totally disagree with R. Gill in his neglecting of the fundamental role which so called “exotic probabilistic models”, in particular, CONTEXTUAL probabilistic models (and, in particular, von Mises frequency model [18]) can play for clarifying the probabilistic structure of quantum theory.

2.1. “Classical” and “quantum” probabilities. If we consider quantum and classical physics from the purely probabilistic viewpoint then we should recognize that there exist two very different probabilistic calculi which are used in completely different situations (moreover, they are developed practically independently):

- 1) *Kolmogorov measure-theoretical model*, 1933, [19].
- 2). *Probabilistic calculus in a Hilbert space*, end of 20th (Born, Jordan, Dirac), e.g., [20].

Remark. It looks very strange (at least for me) that the mathematical formalism (Kolmogorov axiomatics) which describes rigorously classical statistical physics was proposed later than the corresponding quantum formalism...

Since 60th, the main stream of quantum probabilistic investigations was directed to Bell’s model [21] of the EPR-Bohm experiment [22].³ I think that Bell’s considerations gave us a new reformulation of the old problem in probabilistic foundations which was already observed in the two slit experiment. In this experiment the conventional rule for the addition of probabilities is perturbed by the interference term. Already farthers-creators of quantum theory paid large attention to this difference in probabilistic calculi, see, e.g. Dirac [19] or Feynman [27].

We should do something to solve this problem – existence of two different probability calculi – and to unify these models – Kolmogorov and quantum – in some way. I think that such a unification should be done on the basis of some new intuitively clear probabilistic model. We can not start directly

³As pointed out I. Volovich [13] we should sharply distinguish the original EPR model [23] and the EPR-Bohm-Bell model [21]. Recently we proved [24] that for the original EPR model it is possible to construct the local realist representation even by using Kolmogorov-Bell viewpoint to realism, see our further considerations. I also mention investigations of S. Molotkov [25] and I. Volovich [13], [26] who demonstrated the fundamental role of space-time in quantum information theory.

with a Hilbert space. Of course, such a start would be the simplest from the mathematical viewpoint. However, it induced (and continues to induce) terrible misunderstandings, mysteries and prejudices.

I should recall that there already were a few attempts of such a unification, e.g., models of S. Gudder [28] and I. Pitowsky [29]. And these models were well done from the mathematical viewpoint! So formally the problem was solved. The main problem is that those approaches are not intuitively attractive: e.g., nonmeasurable sets in Pitowsky's model or so called influence function in Gudder's model. Well, the origin of the *influence function* is little bit less mysterious than the origin of the Hilbert space. But just little bit...

The same problem was studied in huge number of works in quantum logic, see, e.g., [30]. Quantum logic is also well done from the purely mathematical viewpoint. However, intuitively it is not more attractive than the quantum Hilbert space formalism.

2.2. Frequency approach. I like the frequency model of R. von Mises, [18], because this is the most intuitively attractive probabilistic approach. I would like to remark that the von Mises approach is not at all so bad as it was claimed in 30th, see, e.g., [31], [32], but here I would not like go into mathematical and logical details.

For me one of the main distinguishing features of the von Mises approach is its

CONTEXTUALITY

In this approach a complex of physical conditions (in my papers I propose the terminology - physical context or simply *context*) is represented by a *collective*. R. von Mises underlined: *first a collective then probability*.

In the von Mises model contextualism has the following consequences for the EPR-Bell framework, see [31]:

1) No counterfactual statistical data!

All statistical data are related to concrete collectives (contexts). Here all considerations based on the use of counterfactual arguments (in particular, counterfactual derivations of Bell's inequality, see, e.g., [33], [34]) are nonsense.⁴

2) New viewpoint of independence.

⁴Similar anti-counterfactual conclusions were obtained in some other approaches, e.g., W. De Baere [9], W. De Myunck [10], and K. Hess and W. Philipp [11].

In the frequency model we use a new notion of independence. Not *independence of events* (as in the Kolmogorov model and some other models), but *independence of collectives-contexts*. R. von Mises strongly criticized the conventional notion of independence, namely, event independence. He presented numerous examples in which conventional independence was represented as just a meaningless game with numbers – to obtain factorization of probability into the product of probabilities. In the frequency theory we study independence of collectives (in my terminology – contexts).

If we analyse the well known Bell-Clauser-Horne locality condition by using independence of collectives then we see immediately, see [31], that corresponding collectives are dependent because they contain the same particle-preparation procedure. Hence there are no reasons to suppose the validity of this condition or to connect this condition with locality.

3) Existence of probability distributions for hidden variables.

In the von Mises approach we should start with analysing an experimental situation to be sure that we really have the statistical stabilization (existence of limits) of relative frequencies. In the opposite to Kolmogorov's model we can not start directly with a probability distribution (as Bell did in his Kolmogorov-version of the EPR-arguments). And here a rather strange, but important question arise:

Why do we suppose that a Kolmogorov-probability distribution of hidden variables exists at all?

Well, our experimental equipment produces the statistical stabilization of relative frequencies for observables. Why should it produce the statistical stabilization of relative frequencies for hidden variables? I do not see such reasons... Moreover, intuitively it looks that we can not provide such a statistical stabilization for microsystems: they are too sensible to our macroscopic preparation procedures. In principle, relative frequencies can fluctuate between 0 and 1.⁵

The absence of the statistical stabilization of relative frequencies for mi-

⁵When we say “fluctuate” we have in mind “in the real topology.” So we should remember that all our probabilistic models, e.g., Kolmogorov and von Mises, are rigidly coupled to one very special topology, namely, the real one. Where is the origin of the real topological probability? In the real topology of space-time? Do all our probabilistic models describe just the reality of the real space-time? I discussed these problems at many occasions, see, e.g., [31], [35]. In particular, I developed a p -adic probability model in which reality (p -adic reality) is associated with p -adic collectives-contexts, i.e., “random sequences” for which relative frequencies stabilize in the p -adic topology, [31], [35].

croparameters does not at all contradict to the statistical stabilization of relative frequencies for macro-observables, see examples in [31].

In principle, chaotic fluctuations in the microworld may generate statistical stability on the macrolevel!

Neither it contradicts to realism. But we should distinguish INDIVIDUAL REALISM (physical observables are objective properties of physical systems, i.e., mathematically we can represent them as functions of HV , $a = a(\lambda)$) and KOLMOGOROV-BELL REALISM, namely, the existence of the probability distribution.

Thus von Mises approach strongly differs from Kolmogorov's approach. I disagree with R. Gill who claimed that these approaches give the same consequences: "Regarding to Kolmogorov and von Mises... I do not see any opposition between alternative views of probability here," [16].

2.3. Contextual probability. The frequency approach is really a contextual probabilistic approach. Here we work not with just one fixed collective-context, but with a few collective-contexts by combining probabilities belonging to different collectives-contexts. Of course, R. von Mises did not do so much in this direction. But in any case it was the important step compare with the Kolmogorov approach. Of course, Kolmogorov also paid attention on contextuality of probabilities, see [19], [36]. But his ideology was: for any fixed physical context we should choose a fixed Kolmogorov probability space and work in this space for ever! Even Kolmogorov-conditioning is just event-conditioning in one fixed Kolmogorov probability space – so for a fixed context. R. Gill and I. Helland paid my attention to the fact that statisticians often consider families of Kolmogorov probability spaces depending on some parameter σ . Then by using statistics for random variables they try to find this parameter. However, it is totally different ideology. I think that the main role contextualism of probabilities play when we move from one context to another. In really contextual probability theory we should be able to work not only in one fixed context, but also to describe transformations of probabilities which are induced by context transitions. Such a contextualism can be called *Transition Contextualism* and traditional Kolmogorov (and statistical) contextualism – *Stationary Contextualism*.⁶

⁶There are some probabilistic approaches which are based on conditional probabilities, see, e.g., Renyi [37] or Cox [38]. However, those approaches are still rigidly coupled to Stationary Contextualism. Therefore I disagree with L. Ballentine [1] who claimed that quantum probability can be reduced to such a conditional probability – he used the Cox-model. On the other hand, I appreciate Ballentine's investigations on conditional

I think that one of the main distinguishing features of the quantum probabilistic formalism is the possibility to find dependence of probabilities on contexts. By changing a representation (the orthonormal basis) in a Hilbert space we change context. And the quantum formalism gives us the transformation of probabilities induced by such a context change. Thus quantum probabilistic formalism is a transition-contextual formalism.

At the moment we do not have an intuitively clear transition-contextual probability theory. Therefore I tried to proceed in the von Mises approach. I do not claim that this is the final contextual probabilistic theory. But even by using the von Mises model I could do a lot to clarify the probabilistic structure of quantum theory. As we have already seen, there are at least three contextual arguments against conclusions which J. Bell did on the basis of his Kolmogorov model for the EPR-Bohm experiment: no counterfactual data, no independence of collectives (so the absence of the Bell-Clauser-Horne locality condition), the absence of physical argument supporting Kolmogorov-Bell realism. Moreover, in the frequency model, instead of Bell's (or CHSH) inequality, we obtain some modifications of this inequality, see [40]. In general these modified inequalities do not contradict to experiments. Moreover, recently I derived the quantum EPR-Bohm correlations in the frequency approach (without to appeal to the Hilbert space formalism), see [41].⁷

2.4. Quantum \equiv contextual? My contextual probabilistic investigations demonstrated that many things which are traditionally assigned to the quantum domain can be reproduced by using a contextual probabilistic model. In particular, the interference of probabilities which is typically considered as one of the fundamental features of quantum systems can be obtained in a contextual model, see [43] on the frequency derivation of the interference.

The following question is of the great interest for me:

probabilistic viewpoint of quantum probability. It was the first step – Stationary Contextualism – in the direction of quantum contextualism. I also remark that even earlier L. Accardi provided a detailed analysis of the role of Bayes formula for conditional probability in understanding of quantum probability [39].

⁷Of course, if we are discussing just the Kolmogorov-Bell realism, then all our frequency (contextual) arguments are meaningless. But it seems not to be the case! It seems that when we discuss realism we have in mind INDIVIDUAL REALISM. Such a realism can be described very well by the frequency probability model. I also remark that by using the frequency (contextual) model we can easily resolve the GHZ-paradox, see [42]. Of course, we assume that GHZ were interested to discuss realism and not just the Kolmogorov-Bell realism.

Can “quantum” be totally reduced to “contextual”? Or “quantum” is something more special than “contextual”?

To investigate this question in more detail I considered a formal contextual probability model [5]. I did not try to provide a mathematical description of a context (if it is possible at all in the general case). In such a formal model contexts are just some labels which are assigned to probabilities, $P = P_S$. This is most general framework based on the well acceptable postulate:

All probabilities depend on contexts – complexes of experimental physical conditions.

We can generalize von Mises slogan by saying: first context then probability. In this framework we can reproduce, e.g., the interference of probabilities. However, we immediately see that the formal contextual probability model describes essentially larger set of context transitions than quantum theory. In particular, I found that, besides the trigonometric (“quantum”) interference, there can appear the hyperbolic interference. The latter transformation of probabilities also has a linear representation, namely, in a module over a Clifford algebra [44].

“Theorem”. *Quantum domain is just a proper subset of contextual domain.*

The fundamental question (at least for me) is : Which additional elements should we add to “contextual” to obtain really “quantum”?

Recently in the process of our discussions with I. Volovich we understood that space-time does not present in the conventional axiomatics of quantum mechanics, see, e.g., von Neumann [45]. On the basis of our discussion I. Volovich presented a new system of axioms for quantum mechanics, see [46]. Roughly speaking this is von Neumann’s axiomatics with the additional space-time axiom. However, in the process of further discussions we understood that space-time is nothing than a special context – *space-time context*, see our quant-preprint [47]. According to the general ideology of the contextual approach probabilities should also depend on the space-time context. I do not think that space-time as itself is the “fundamental quantum element.” The same space-time we also use in classical mechanics.

3 “Fundamental quantum element”

The contextual probabilistic investigations demonstrated that a rather special behaviour of quantum probabilities is, in fact, nothing than contextual behaviour. Thus the Hilbert space calculus of probabilities is not the “fundamental quantum element”. The fundamental is the very special form of time-evolution of probability distributions, namely, *Schrödinger evolution*. I do not afraid to say that the real quantum mechanics was discovered by E. Schrödinger [48] and not by W. Heisenberg [49]. In particular, I do not consider *noncommutativity* as a fundamental really quantum feature. The non-commutative representation of physical observables in conventional quantum theory is just a sign of contextuality. In principle, such a representation can arise in various classical physical models. Such examples were presented in our paper with S. Kozyrev [50]. I think that there can be found hundreds of noncommutative classical models. In particular, Heisenberg’s *uncertainty relations* are just contextual statistical relations describing dependence of dispersions on physical contexts. From this point of view *quantum logic* is, in fact, not quantum, but a *contextual logic*.

The main quantum mystery is

MYSTERY of SCHRÖDINGER REPRESENTATION.

I would like to thank L. Ballentine, K. Valiev, S. Albeverio, E. Beltrametti, L. Accardi, E. Loubenets, R. Gill, T. Hida, D. Greenberger, S. Gudder, W. De Muynck, J. Summhammer, P. Lahti, J-A. Larsson, H. Atmanspacher, B. Coecke, S. Aerts, A. Peres, A. Holevo, L. Polley, A. Zeilinger, C Fuchs, L. Hardy, A. Plotnitsky, A. Shimony, R. Jozsa, J. Bub, C. Caves, K. Gustafsson, H. Bernstein, for fruitful (and rather critical) discussions on foundations of quantum mechanics.

I would like to thank B. Hiley for discussions on the role of space in quantum formalism (in the relation to the Bohmian mechnaics), S. Molotkov for discussions on the role of space-time in quantum information, and I. Volovich for discussions on the space-time axiom of quantum theory.

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